CHAPTER 2

Number Systems

(Solutions to Odd-Numbered Problems)

Review Questions

1. A number system shows how a number can be represented using distinct symbols.
3. The base (or radix) is the total number of symbols used in a positional number system.
5. The binary system is a positional number system that uses two symbols (0 and 1) to represent a number. The word binary is derived from the Latin root *bini* (two by two) or *binarius* (related to two). In the binary system, the base is 2.
7. The hexadecimal system is a positional number system with sixteen symbols. The word hexadecimal is derived from the Greek root *hex* (six) and the Latin root *decem* (ten). To be consistent with decimal and binary, it should have been called *sexadecimal*, from Latin roots *sex* and *decem*. In the hexadecimal system, the base is 16.
9. Four bits in binary is one hexadecimal digit.

Multiple-Choice Questions

11. c 13. b 15. a 17. b 19. c 21. b

Exercises

23.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(01101)_2$			0	8	4	0	1				13
$(1011000)_2$	64	0	16	8	0	0	0				88
$(011110.01)_2$		0	16	8	4	2	0	0	1/4		30.25
$(111111.111)_2$		32	16	8	4	2	1	1/2	1/4	1/8	63.875

25.

Place values	512	64	8	1	1/8	1/64	
$(237)_8$ =		+ 2×64	+ 3×8	+ 7×1	+	+	= 159
$(2731)_8$ =	2×512	+ 7×64	+ 3×8	+ 1×1	+	+	= 1497
$(617)_8$ =		+ 6×64	+ 1×8	+ 7×1	+ $7 \times 1/8$	+	= 399.875
$(21.11)_8$ =		+	+ 2×8	+ 1×1	+ $1 \times 1/8$	+ $1 \times 1/64$	≈ 17.141

27.

a. $1156 = (2204)_8$ as shown below:

0 ←	2 ←	18 ←	144 ←	1156
	↓	↓	↓	↓
	2	2	0	4

b. $99 = (134)_8$ as shown below:

0 ←	1 ←	12 ←	99
	↓	↓	↓
	1	4	3

c. $11.4 = (13.3146)_8$ as shown below:

0 ←	1 ←	11		.4 →	.2 →	.6 →	.8 →	4
	↓	↓		↓	↓	↓	↓	
	1	3	•	3	1	4	6	

d. $72.8 = (110.6314)_8$ as shown below:

0 ←	1 ←	9 ←	72		.8 →	.4 →	.2 →	.6 →	.8
	↓	↓	↓		↓	↓	↓	↓	
	1	1	0	•	6	3	1	4	

29.

$(514)_8$ =		101	001	100			=	1	0100	1100			= $(14C)_{16}$
$(411)_8$ =		100	001	001			=	1	0000	1001			= $(109)_{16}$
$(13.7)_8$ =			001	111	•	111	=		00	1011	•	1110	= $(B.E)_{16}$
$(1256)_8$ =	001	010	101	110			=	0010	0101	1110			= $(25E)_{16}$

31.

$(01101)_2$	=		001	101			=	**$(15)_8$**
$(1011000)_2$	=	001	011	000			=	**$(130)_8$**
$(011110.01)_2$	=		011	110	•	010	=	**$(36.2)_8$**
$(111111.111)_2$	=		111	111	•	111	=	**$(77.7)_8$**

33.

121	=	0	+	64	+	32	+	16	+	8	+	0	+	0	+	1	=	**$(01111001)_2$**
78	=	0	+	64	+	0	+	0	+	8	+	4	+	2	+	0	=	**$(01001110)_2$**
255	=	128	+	64	+	32	+	16	+	8	+	4	+	2	+	1	=	**$(11111111)_2$**
214	=	128	+	64	+	0	+	16	+	0	+	4	+	2	+	0	=	**$(11010110)_2$**

35.

a. binary: $2^6 - 1 = 63$

b. decimal: $10^6 - 1 = 999{,}999$

c. hexadecimal: $16^6 - 1 = 16{,}777{,}215$

d. octal: $8^6 - 1 = 262{,}143$

37.

a. $\lceil 5 \times (\log 2) / (\log 10) \rceil = \lceil 16.6 \rceil = 2$

b. $\lceil 3 \times (\log 8) / (\log 10) \rceil = \lceil 16.6 \rceil = 3$

c. $\lceil 3 \times (\log 16) / (\log 10) \rceil = \lceil 16.6 \rceil = 4$

39. Using the result of previous exercise, we can find the equivalent as:

a. $7.1875 = (111)_2 + (0.001)_2 + (0.0001)_2 = (111.0011)_2$

b. $12.540625 = (1100)_2 + (0.1)_2 + (0.001)_2 + (0.000001)_2 = (1100.101001)_2$

c. $11.40625 = (1011)_2 + (0.01)_2 + (0.001)_2 + (0.00001)_2 = (1011.01101)_2$

d. $0.375 = (0.01)_2 + (0.001)_2 = (0.011)_2$

41.

a. $\lceil \log_2 1000 \rceil = \lceil \log 1000 / \log 2 \rceil = \lceil 9.97 \rceil = 10$

b. $\lceil \log_2 100{,}000 \rceil = \lceil \log 100{,}000 / \log 2 \rceil = \lceil 16.6 \rceil = 17$

c. $\lceil \log_2 64 \rceil = \lceil \log_2 2^6 \rceil = \lceil 6 \times \log_2 2 \rceil = \lceil 6 \rceil = 6$

d. $\lceil \log_2 256 \rceil = \lceil \log_2 2^8 \rceil = \lceil 8 \times \log_2 2 \rceil = \lceil 8 \rceil = 8$

43.

a.	17×256^3	+	234×256^2	+	34×256^1	+	14×256^0	=	300,556,814
b.	14×256^3	+	56×256^2	+	234×256^1	+	56×256^0	=	238,611,000
c.	110×256^3	+	14×256^2	+	56×256^1	+	78×256^0	=	1,864,425,678
d.	24×256^3	+	56×256^2	+	13×256^1	+	11×256^0	=	406,326,539

45.

a. 15

b. 27

c. This is not a valid Roman Numeral (V cannot come before L)

d. 1157

47.

a. Not valid because I cannot come before M

b. Not valid because I cannot come before C

c. Not valid because V cannot come before C

d. Not valid because 5 is written as V not VX

49.

a. First, we convert the three numbers to base 60 as shown below:

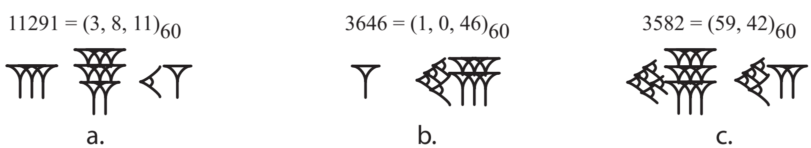
0	←	3	←	188	←	11291
		↓		↓		↓
		3		**8**		**11**

0	←	1	←	60	←	3646
		↓		↓		↓
		1		**0**		**46**

0	←	59	←	3582
		↓		↓
		59		**42**

The equivalent Babylonian numerals are shown in Figure S2.49

Figure S2.49 *Exercise 49*



b. In Babylonian numerals, they used extra space when a zero was needed in the middle of the number. When a zero was need at left, they did not use anything; They probably recognized it from the context.