

國立臺灣師範大學 98 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系(組)：資訊工程學系

注意：1.本試題共 2 頁，請依序作答，並標明題號，不必抄題。

2.答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

1. (10 分) Find the line tangent to the right-hand hyperbola branch defined parametrically by

$$x = \sec t, \quad y = \tan t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2} \quad \text{at point } (\sqrt{2}, 1), \text{ where } t = \pi/4.$$

2. (12 分) A police cruiser(警車), approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 km north of the intersection and the car is 0.8 km to the east, the police determine with radar that the distance between them and the car is increasing at 28 km/hour. If the cruiser is moving at 60 km/hour at the instant of measurement, what is the speed of the car? (See Figure 1)

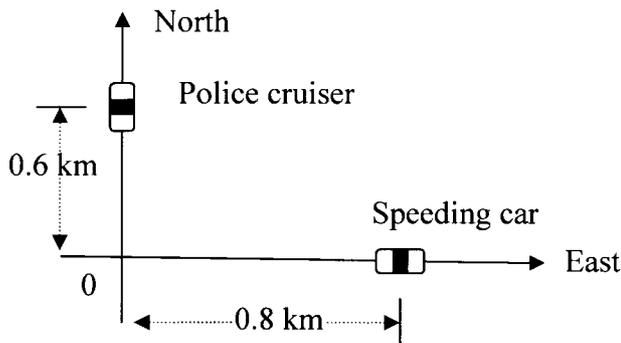


Figure 1.

3. (10 分) How close does the curve $y = \sqrt{x}$ come to the point $(3/2, 0)$? (That is, find the minimum distance between the curve and the point. See Figure 2.)

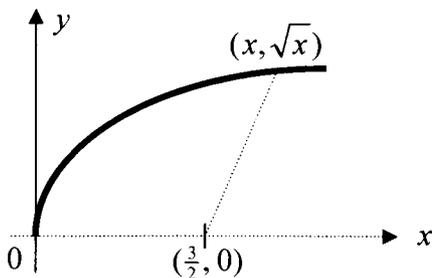


Figure 2.

4. (a) (10 分) Find the length of the curve $y = \frac{(x^2 + 2)^{3/2}}{3}$ from $x = 0$ to $x = 3$.
- (b) (10 分) Find the length of the curve given by the polar coordinate equation:
 $r = 8\sin^3(\theta/3)$, $0 \leq \theta \leq \pi/4$.
5. (a) (8 分) Solve the differential equation $2y' = e^{x/2} + y$.
- (b) (8 分) Solve the differential equation $\frac{dy}{dx} = x^2 e^{4x}$.
- (c) (8 分) Find the solution to $\frac{dy}{dx} = 2xy(y^2 + 1)$ that satisfies $y(0) = 1$.
6. (a) (6 分) Find $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.
- (b) (6 分) Find $\lim_{x \rightarrow 0} (e^x + x)^{1/x}$.
- (c) (6 分) Find $\int_{-\infty}^{+\infty} f(x) dx$ where $f(x) = 3e^{(x-e^x)}$.
- (d) (6 分) Find $\lim_{n \rightarrow \infty} a_n$ where $a_n = \left(\frac{n+1}{n-1} \right)^n$.