國立臺灣師範大學95學年度學士班二年級轉學生招生考試試題

學系(組)：資訊工程學系

專門科目：微積分

注意事項：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

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1. (12 pts) Find the Taylor series generated by $f(x)=1/x$ at $a=3$. Where, if anywhere, does the series converge to $1/x$?

2. (10 pts) Evaluate $\int e^x \cos x\, dx$.

3. (12 pts) Find the solution to $dy/dx = 2xy(y^2+1)$ that satisfies $y(0)=1$.

4. (12 pts) Find (a) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$; (b) $\lim_{x\to\infty}\frac{3}{x\ln x}\int_1^x \ln t\, dt$;
(c) $\lim_{k\to\infty} 6(1+\frac{r}{k})^{kt}$.

5. (10 pts) Find the values of p for which each integral converges:
(a) $\int_1^2 \frac{dx}{x(\ln x)^p}$; (b) $\int_2^\infty \frac{dx}{x(\ln x)^p}$.

6. (12 pts) The usual way to evaluate the improper integral $I=\int_0^\infty e^{-x^2}dx$ is first to calculate its square: $I^2=\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$. Evaluate the integral and solve the resulting equation for I.

7. (10 pts) Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at point $(x,y,z)=(1, 1, 1)$ and $z^3-xy+yz+y^3-2=0$.

8. (10 pts) Find the tangent and normal to the curve $x^3+y^3-9xy=0$ at point (2, 4).

9. (12 pts) Find the horizontal tangents to the graph of the cardioid $r=1-\cos\theta$, $0\le\theta\le 2\pi$. (That is , find the points at which horizontal lines tangent to the curve.)