國立臺灣師範大學九十五學年度碩士班考試入學招生試題

數學基礎 科試題（資訊工程研究所用，本試題共 **3** 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. Determine which ones in the following languages are accepted by the finite-state automata shown in Figure 1. (5 分)

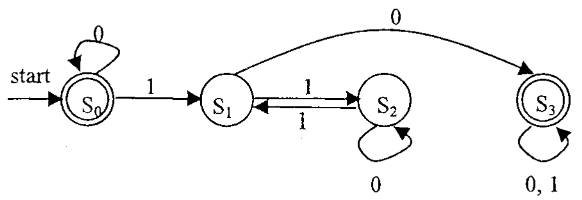

Figure 1

a) $\{0^m(101)^n | m \geq 1, n \geq 0\}$
b) $\{(0111)^n | n \geq 0\}$
c) $\{1^m0^n | m \geq 0, n \geq 0\}$
d) $\{(010)^m | m \geq 1\}$
e) $\{0^n1^m0 | n \geq 0, m \geq 1\}$

2. Determine whether each of the logical implication is always true. (6 分)

a) $p \to (p \wedge q)$
b) $(p \vee q) \to q$
c) $[(p \vee q) \wedge \neg q] \to p$
d) $[(p \to q) \wedge \neg q] \to \neg p$

3. An edge coloring of a graph is an assignment of colors to edges so that edges incident with a common vertex are assigned different colors. The edge chromatic number of a graph is the smallest number of colors that can be used in an edge coloring of the graph. Please find the edge chromatic numbers of the following graphs, respectively. (6 分)

a) K_n
b) $K_{m,n}$
c) C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, …, $\{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$.
d) W_n, $n \geq 3$, obtained by adding an additional vertex to the cycle C_n, and connecting this new vertex to each of the n vertices in C_n by new edges.

請翻頁繼續作答

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4. Use mathematical induction to prove that every amount of postage of 12-dollor or more can be found using just 4-dollar and 5-dollar stamps. (8 分)

5. How many integers in a sequence of 101 different integers will form the largest increasing or decreasing subsequence of the sequence? Please explain your answer. (6 分)

6. The problem of derangements:

   a) How many derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with 1, 2, 3, and 4 in some order? (3 分)

   b) How many derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with 5, 6, 7, and 8 in some order? (3 分)

7. Please find the coefficient of x^{15} term in $(x^3 - 5x)/(1-x)^3$. (5 分)

8. Let A = {w, x, y, z}

   a) Please determine the number of relations on A that are reflexive and symmetric. (4 分)

   b) Determine whether each of the following statements is true or false. (4 分)

      i) If R is a relation on A and $|R| \geq 4$, then R is reflexive.

      ii) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 symmetric $\Rightarrow R_1$ symmetric.

      iii) If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 antisymmetric $\Rightarrow R_1$ antisymmetric.

      iv) If R is an equivalent relation on A, then $4 \leq |R| \leq 16$.

9. Given two vectors $A = (1,2,3,4,5)$ and $B = (1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5})$ in 5-dimensional space $\boldsymbol{R}^5$. Find two vectors C and D in $\boldsymbol{R}^5$ satisfying the following three conditions: C is parallel to A, D is orthogonal to A, and $C + D = B$. (6 pts)

10. The square matrix $A(t)$ has its element $a_{ij}(t)$ being a function of t. Denote $A'(t) = \frac{d}{dt}A(t)$, the derivative of $A(t)$. Suppose that for a t-value at which $A(t)$ is differentiable and possesses an inverse, denoted by $A^{-1}(t)$. Find the derivative of $A^{-1}(t)$, expressed in terms of $A'(t)$ and $A^{-1}(t)$; that is, find $\frac{d}{dt}A^{-1}(t) = ?$ (6 pts)

11. Determine a,b,c,d,e,f, given that the vectors (1,2,3), (1,0,-1), and (1,-1,0) are eigenvectors of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ d & e & f \end{bmatrix}$. (6 pts)

12. For a 3x3 matrix $A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$,

   i. Find A^{-1}. (4 pts)
   ii. Show that A is an orthogonal matrix. (4 pts)
   iii. Is matrix A (a) positive definite, (b) positive semidefinite, (c) negative definite, or (d) negative semidefinite? (4 pts)

13. Given $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

   i. Find the characteristic values and a normalized modal matrix corresponding to matrix A. (6 pts)
   ii. Find e^A. (8 pts)

14. If $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$, find a nonsingular matrix P such that $P^{-1}AP = \begin{bmatrix} 6 & 0 \\ 0 & -1 \end{bmatrix}$, and $P\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. (6 pts)