國立臺灣師範大學 98 學年度碩士班招生考試試題

科目：**數學基礎** 適用系所：**資訊工程學系**

注意：1.本試題共 4 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (3 分) How many elements does the power set $P(\{\varnothing, a, \{a\}, \{\{a\}\}\})$ have?

2. (6 分) Determine whether each of these statements is true or false.

   (a) $\{0\} \subset \varnothing$, (b) $\{0\} \in \{0\}$, (c) $\{\varnothing\} \subseteq \{\varnothing\}$.

3. (6 分) Find the values for (a) $\left\lceil -\frac{3}{4} \right\rceil$, (b) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$, (c) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$.

4. (6 分) Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \cdots$,

   if (a) $a_n = 6n$ and (b) $a_n = 10^n$.

5. (4 分)How many ways are there to place 10 indistinguishable balls into 4 distinguishable bins?

6. (6 分) Define the Boolean operators $\wedge$ and $\vee$, which operate on pairs of bits, as

   $$b_1 \wedge b_2 = \begin{cases} 1 & \text{if } b_1 = b_2 = 1 \\ 0 & \text{otherwise} \end{cases}, \qquad b_1 \vee b_2 = \begin{cases} 1 & \text{if } b_1 = 1 \text{ or } b_2 = 1 \\ 0 & \text{otherwise} \end{cases}.$$

   A matrix with entries that are either 0 or 1 is called a 0-1 matrix. Let $A = [a_{ij}]_{m \times k}$ and $B = [b_{ij}]_{k \times n}$ be two 0-1 matrices. Their Boolean product is defined as $C = A \otimes B$, where $C = [c_{ij}]_{m \times n}$ and $c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \cdots \vee (a_{ik} \wedge b_{kj})$.

   (a) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Compute $A \otimes B$.,

   (b) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Give a solution for the equation $A \otimes X = B$.

7. (4 分) Find all the roots of the characteristic equation of the difference equation $x_n - 4x_{n-1} + 3x_{n-2} = 0$.

8. (3 分) Suppose A is an $n \times n$ matrix and $\boldsymbol{x}$ is an $n \times 1$ vector. Give the condition that the length of $A\boldsymbol{x}$ equals the length of $A^T \boldsymbol{x}$.

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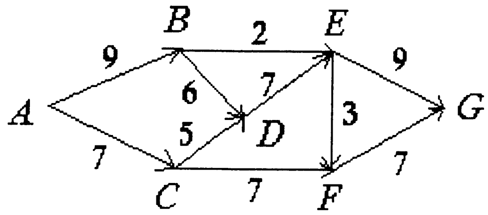
9. (6 分) Is the language (a) $L_1 = \{a^n b^n \mid n = 1, 2, 3, \cdots\}$, (b) $L_2 = \{a^n b^n c^n \mid n = 1, 2, 3, \cdots\}$, (c) $L_3 = \{b^n a b^m \mid n = 0, 1, 2, \cdots, m = 1, 2, 3, \cdots\}$, a regular, context-free or context-sensitive language?

10. (6 分) Give the capacities of (a) cut ($\{A, C, F\}$, $\{B, D, E, G\}$) and

(b) cut ($\{A, D, E\}$, $\{B, C, F, G\}$) for the following network.



11. (4 分) Given that A= $\begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$ and det(A)=4, please evaluate the determinants for the following matrices.

(a) $\begin{bmatrix} -r & -s & -t \\ 2x-4r & 2y-4s & 2z-4t \\ 3u & 3v & 3w \end{bmatrix}$

(b) $(3A^{-1})^T$

12. (4 分) There are four linear transformations defined by the following equations. Which of the linear transformations is one-to-one? (maybe more than one)

(a) T: $R^2 \to R^2$ $\quad w_1 = x_1 + x_2$

$w_2 = 6x_1 + 7x_2$

(b) T: $R^2 \to R^2$ $\quad w_1 = 3x_1 - 9x_2$

$w_2 = 4x_1 - 12x_2$

(c) T: $R^2 \to R^2$ $\quad w_1 = 2x_2$

$w_2 = 2x_1$

(d) T: $R^3 \to R^3$ $\quad w_1 = x_1 - 7x_2 + 7x_3$

$w_2 = -2x_1 + 2x_2 + x_3$

$w_3 = 3x_1 - 5x_2 + x_3$

13. (4 分) Which of the following statements is correct? (maybe more than one)

(a) The vectors (5, -1, 6), (2, 4, 1), and (0, 1, 1) in R^3 lie in a plane.

(b) The vectors (1, 1, 3), (2, 0, 5), and (1, -3, 1) in R^3 lie in a plane.

(c) The vectors (1, -7, 4), (2, -14, 8), and (-3, 21, -12) lie in R^3 lie in the same line.

(d) The vectors (1, 6, 5), (2, 12, 11), and (3, 18, 15) lie in R^3 lie in the same line.

14. (4 分) Which of the sets of vectors is a basis for R^3? (maybe more than one)

(a) (1, 1, 1), (2, 2, 0), (3, 0, 0)

(b) (0, 17, 8), (1, 7, 5), (2, -3, 2)

(c) (2, 5, 2), (1, -1, 3), (0, 1, 1)

(d) $(\sqrt{2},0,\sqrt{2}),(\sqrt{3},0,\sqrt{6}),(0,-\sqrt{6},\sqrt{3})$

15. (4 分，請列計算過程) Please solve the following system of equations by using Cramer's Rule.

$$x+y-z=2$$

$$3x-y+z=5$$

$$3x+2y+4z=0$$

16. (5 分，請列計算過程) Consider R^3 with the Euclidean inner product. Please use the Gram-Schmidt process to transform the given basis {(1, 1, 1), (1, 1, 0), (1, 0, 0)} into an orthogonal basis.

17. (5 分，請列計算過程) Please find the orthogonal projection of u on span $\{v_1, v_2\}$, where $u = (1, 3, -2)$, $v_1=(1, 0, 3)$, and $v_2=(1, 1, 2)$.

18. (請列計算過程) Consider the bases $B=\{u_1, u_2\}$ and $B'=\{v_1, v_2\}$ for R^2, where

$$u_1=\begin{bmatrix}3\\3\end{bmatrix},\ u_2=\begin{bmatrix}-2\\1\end{bmatrix},\ v_1=\begin{bmatrix}2\\5\end{bmatrix},\ v_2=\begin{bmatrix}3\\7\end{bmatrix}$$

(a) (3 分) Please find the transition matrix $P_{B,B'}$ from B' to B.

(b) (3 分) Please compute the coordinate matrix $[w]_{B'}$ where $w=\begin{bmatrix}4\\7\end{bmatrix}$.

(c) (2 分) Use your answers of parts (a) and (b) to compute $[w]_B$.

19. (請列計算過程)Consider the linear operator T: $R^2 \to R^2$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ 2x_1 + x_2 \end{bmatrix}$.

(a) (4 分) Please determine the eigenvalues of T.

(b) (4 分) Please determine a basis B' such that $[T]_{B'}$ is diagonal.

(c) (4 分) Use $[T]_{B'}$ to compute $T^5\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)$.