國立臺灣師範大學資訊工程學系
九十九學年度第一學期
博士班資格考

考試科目：演算法
總分一百分
請在答案卷作答，在題目卷上作答不予計分

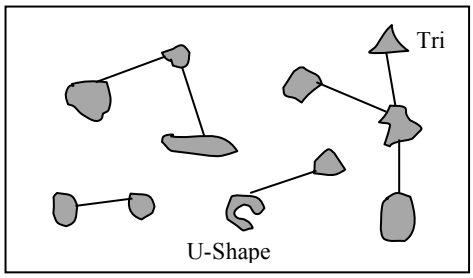
1. Given the input 30, 20, 56, 75, 31, 19 and a hash function $h(K) = K \bmod 11$.
   (a) **(5 pts)** Construct the hash table; resolve the collision by chaining.
   (b) **(5 pts)** Find the *average* number of key comparisons in a successful search in this table.

2. **(10 pts)** A sorting algorithm is *stable* if numbers with the same value appear in the output array in the same order as they do in the input array. However, not all of the sorting algorithms are stable. Give a scheme that makes any sorting algorithm stable.

3. The 0-1 knapsack problem is the following. A thief robbing a store finds n items. The i-th item is worth v_i dollars and weighs w_i pounds, where v_i and w_i are integers. The thief wants to take as valuable a load as possible, but he can carry at most W pounds in his knapsack, for some integer W. Which item(s) should he take?
   (a) **(5 pts)** A simple greedy method is to take the items with the largest value first. Please give an example to show that this method may not give an optimal solution.
   (b) **(5 pts)** Another simple greedy method is to take the items with the lightest weights first. Please give an example to show that this method may not give an optimal solution.
   (c) **(10 pts)** If we take the items with the largest value per pound, does the greedy method give an optimal solution? Please explain the reasoning of your answer.
   (d) **(10 pts)** Suppose we can take fractions of items; that is, we do not have to make a binary (0-1) choice for each item. For example, we can take 0.5 of the item 1, and 0.3 of the item 2. Does the greedy method that takes the items with the largest value per pound give an optimal solution? Please explain the reasoning of your answer.

4. Tony is planning to attend a series of Computer Science courses to raise his career competence. There are precedence constraints between some courses; for example, "Basic Programming" is a prerequisite of "Data Structures." Tony does not want to quit his job, and he will attend one course at a time. He is thinking about how to arrange the order of courses so that when he starts a new course he has already finished all prerequisites.
   (a) **(2 pts)** The order-determining problem is close to a typical problem in the graph theory. What's the *name* of this problem?
   (b) **(5 pts)** Can you explain how to *solve the problem efficiently*?
   (c) **(3 pts)** Can you analyze the *time complexity* of you algorithm? Give the data structures used in the algorithm if they have impact on the complexity.

5. Islandy is a country consisting of many islands. Each island is surrounded by the sea. In the past years the Islandy government has constructed a lot of bridges for his people to travel between the islands. (All bridges are bidirectional.) Although the government has worked hard and many bridges have been built, the Islandy people still cannot travel around the whole country through the bridges. (For example, the islanders of Tri cannot travel to the U-Shape Island through bridges.)



   (a) **(5 pts)** The Islandy economics are not good, and the government is thinking about connecting the entire country (i.e. each island is connected to any other with at least one path) with the minimum number of bridges. Could you please give an *efficient* algorithm to determine the number and locations of new bridges? Analyze the time complexity also. Give the data structures used in the algorithm if they have impact on the complexity.

   (b) **(10 pts)** Since the costs of bridge between islands are different, the Islandy government soon notices that the "minimum number of bridges" is not a good objective. Instead, the government wants to connect the whole country with "the minimum cost." Given the construction cost of bridge between every two unconnected islands, please give an *efficient* algorithm to determine the locations of these bridges. Analyze the time complexity also. Give the data structures used in the algorithm if they have impact on the complexity.

6. The Dijkstra's algorithm is an algorithm for the single-source shortest-paths problem. It is a greedy algorithm, which repeatedly selects the unsolved vertex with the minimum shortest-path estimate and relaxes all leaving edges. The above two operations, Extract-Min and Relax, are the two mainly determining the time complexity of the Dijkstra's algorithm.
   (a) **(5 pts)** If we do the two main operations based on a linear array, analyze the time complexity.
   (b) **(5 pts)** If we do the two main operations based on a binary heap, analyze the time complexity.
   (c) **(5 pts)** Give a condition where the Dijkstra's algorithm is not applicable. Which algorithm will you recommend in that condition?

7. The Floyd-Warshall algorithm solves the all-pairs shortest paths problem with time complexity $\theta(n^3)$. The pseudo code is as follows:

```
Floyd-Warshall(W)
1  n ← rows[W]
2  D^(0) ← W
3  for k ← 1 to n
4     do for i ← 1 to n
5        do for j ← 1 to n
6           d_ij^(k) ← min(d_ij^(k–1), d_ik^(k–1) + d_kj^(k–1))
7  return D^(n)
```

   (a) **(5 pts)** Please explain the meaning of $d_{ij}^{(k)}$.
   (b) **(5 pts)** Given a directed graph $G(V, E)$, the transitive closure of G is defined as the graph $G^* = (V, E^*)$, where $E^* = \{(i, j)$: there is a path from vertex i to vertex j in $G\}$. Please give an algorithm for computing the transitive closure based on the Floyd-Warshall algorithm.