國立臺灣師範大學資訊工程學系
九十九學年度第二學期
博士班資格考

考試科目：離散數學
總分一百分
請在答案卷作答，在題目卷上作答不予計分

1. (10pts) Show that $\neg(p \vee q)$ is not equivalent to $\neg p \vee \neg q$. Note: Do not use logical equivalences such as $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

2. (10pts) Prove that $2^n > n^2$ if n is an integer greater than 4.

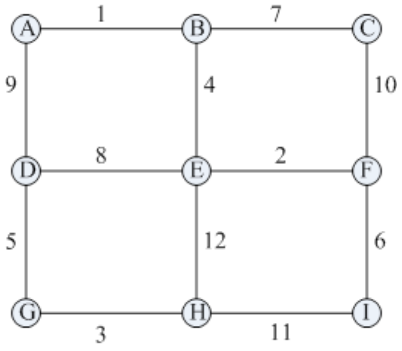
3. (10pts) Let N be the set of natural numbers {0, 1, 2, 3, …} and Z the set of integers {…, -2, -1, 0, 1, 2, …}. Let $f : \mathrm{N} \to \mathrm{Z}$ be defined by the two-part rule $f(n) = \begin{cases} n/2, & \text{if } n \text{ is even;} \\ -(n+1)/2, & \text{if } n \text{ is odd.} \end{cases}$ Determine whether f is onto Z.

4. (10pts) Is it true that x^3 is $O(7x^2)$? Please prove or justify your answer.

5. (10pts) What is the most efficient way to multiply the matrices A_1, A_2, A_3, and A_4 if the dimensions of these matrices are 10×2, 2×5, 5×20,and 20×3, respectively?

6. (16pts) Please give an example of a simple graph that
   (a) have both Eulerian and Hamiltonian circuits;
   (b) have a Hamitonian circuit, but no Eulerian circuits;
   (c) have an Eulerian circuit, but no Hamiltonian circuits;
   (d) have neither Eulerian and Hamitonian circuits.

7. (14pts) Given the graph to the right, please show the step-by-step results of finding its minimum spanning tree using the ***Prim's Algorithm*** and ***Kruskal's Algorithm*** respectively.



8. (10pts) Among the n-digit k-nary sequences, how many of them have an **even** number of 0's? (hint: including the cases that there are zero 0's in the sequence)

9. (10pts) While on a 30-day vacation, Rafael will play at least one set of tennis each day, but he won't play more than 50 sets total during this time. Please prove that no matter how he distributes his sets during the 30 days, there is a consecutive span of days during which he will play exactly 9 sets.